Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 9, 2004*
Revised: *November 8, 2004*
Accepted: *November 22, 2004*

Racetrack inflation

Jose J. Blanco-Pillado,[a] Clifford P. Burgess,[b] James M. Cline,[b] Cristina Escoda,[c] Marta Gómez-Reino,[d] Renata Kallosh,[e] Andrei D. Linde[e] and Fernando Quevedo[c]

[a] *Center for Cosmology and Particle Physics*
New York University, New York, NY 10003, U.S.A.
[b] *Physics Department, McGill University, 3600 University Street*
Montréal, Québec, H3A 2T8, Canada
[c] *DAMTP, Centre for Mathematical Sciences*
University of Cambridge, Cambridge CB3 0WA, U.K.
[d] *Martin Fisher School of Physics*
Brandeis University, Waltham MA 02454, U.S.A.
[e] *Department of Physics, Stanford University*
Stanford, CA 94305–4060, U.S.A.
E-mail: bp442@nyu.edu, cliff@physics.mcgill.ca, jcline@physics.mcgill.ca, c.escoda@damtp.cam.ac.uk, marta@brandeis.edu, kallosh@stanford.edu, alinde@stanford.ed, f.quevedo@damtp.cam.ac.uk

Abstract: We develop a model of eternal topological inflation using a racetrack potential within the context of type-IIB string theory with KKLT volume stabilization. The inflaton field is the imaginary part of the Kähler structure modulus, which is an axion-like field in the 4D effective field theory. This model does not require moving branes, and in this sense it is simpler than other models of string theory inflation. Contrary to single-exponential models, the structure of the potential in this example allows for the existence of saddle points between two degenerate local minima for which the slow-roll conditions can be satisfied in a particular range of parameter space. We conjecture that this type of inflation should be present in more general realizations of the modular landscape. We also consider 'irrational' models having a dense set of minima, and discuss their possible relevance for the cosmological constant problem.

Keywords: D-branes, Superstring Vacua, Supersymmetry Phenomenology.

Contents

1. Introduction

The past three years have seen a revival of attempts to derive cosmological inflation from string theory. On the theoretical side, the main reason for this effort has been the development of tools for studying modulus stabilization, through the explicit construction of nontrivial potentials for moduli fields within string theory. From the observational side, the impetus has been the successful inflationary description of recent CMB results.

Early attempts to find inflation within string theory looked to the string dilaton S and the geometrical moduli fields (such as the volume modulus, T) as natural candidates for the inflaton field [1, 2]. However these attempts were largely thwarted by the flatness (to all orders in perturbation theory) of the corresponding scalar potentials, together with the discovery that the few calculable nonperturbative potentials considered [3]–[5] were not flat enough to satisfy the slow-roll conditions needed for inflation. This led to the alternative proposal of D-brane inflation, which instead considered the separation between branes as an inflaton candidate [6]–[12].

In the absence of an understanding of how moduli are fixed, the main working assumption used when exploring these proposals was simply that all moduli aside from the putative inflaton were fixed by an unknown mechanism at large scales compared with those relevant to inflation. In particular it was assumed that this fixing could be ignored when

JHEP11(2004)063

analyzing the inflaton dynamics. It is now possible to do better than this, following the recent study of brane/antibrane inflation [13] (see also [14]–[16]) which could follow the interplay between the inflaton and other moduli by working within the KKLT scenario for moduli fixing [17].

In so doing these authors discovered an obstacle to the successful realization of inflation, which is a version of the well-known η problem of F-term inflation within supersymmetric theories [18]. The problem arises because the structure of the supergravity potential (see e.g. eq. (3.4)) involves an overall factor proportional to e^K where K is the Kähler potential, and this naturally induces a mass term which is of order the Hubble scale, H, for the inflaton field. As such it contributes a factor of order one to the slow-roll parameter η, which must be very small in order for the model to agree with observations. Consequently, a fine tuning of roughly 1 part in 100 in [13] and in 1 part in 1000 in [15] is required in these models for a successful inflation. This tuning would be unnecessary if the inflaton field did not appear within the Kähler potential. Unfortunately K does depend on the inflaton in the D3/anti-D3 systems considered in [13, 15] due to the absence of isometries in Calabi-Yau spaces. A brane position can avoid appearing within the Kähler potential as well as non-perturbative superpotential for the system of D3/D7 branes [10]. This is due to the underlying $\mathcal{N} = 2$ supersymmetry of these systems, and therefore for these model there is no η problem [14].

In this article we present a different approach for string inflation for which a geometrical modulus is the inflaton, without the need of introducing the interacting D-branes and their separation. In this way we revive the original proposals for modular inflation of [1, 2] (see also [9]). Our proposal is based on a simple extension of the KKLT scenario to include a racetrack-type superpotential, along the lines developed in [19]. The difficulty with obtaining inflation using the simplest potentials considered in the past [3]–[5] — as well as more recently in [17] — is that the potential is never flat enough to allow for slow roll. However in nonperturbative potentials of the modified racetrack[1] type such as arise in type IIB string compactifications we find saddle points which give rise precisely to the conditions for topological inflation [20, 21], being flat enough to provide the right number of e-foldings and the flat spectrum of the perturbations of the metric. An attractive feature of using a modulus as inflaton is that the tree-level Kähler potential can be independent of some fields, like the imaginary part of the Kähler modulus field Im T, and so at this level the η problem is not necessarily present. (However approximate symmetries such as shifts in Im T are typically broken both by the nonperturbative superpotential and by loop corrections to the Kähler potential.) We are led in this way to an inflaton field which is an axion-like pseudo-Goldstone mode. In this respect, our scenario resembles the natural inflation scenario [22].

A feature of natural inflation scenarios which is *not* shared by the inflation which we find is the assumption that only the axion field evolves, with the pseudo-Goldstone mode running along the valley of the potential for which all other fields are stabilized. We find no such regime so far in supergravity or string theory, and instead in our scenario we find

[1]By 'modified racetrack' we mean a racetrack superpotential [4] — i.e. containing more than one exponential of the Kähler modulus — which is modified by adding a constant term [17, 19], such as arises from the three-form fluxes of the type-IIB compactifications.

that the volume modulus, Re T, is stabilized only in the vicinity of the KKLT minima and near a saddle point of the potential in between these minima, where the axion field vanishes. Inflation does not occur near the KKLT minima but, as we shall see, under certain conditions it can occur near the saddle points.

We start, in section 2, with a short recapitulation of topological inflation [20, 21] before discussing our main results about racetrack inflation in section 3. In section 4 we present an interesting generalization of our results to include an 'irrational' dependence on the inflaton field leading to an infinite number of vacua, and we discuss there its possible implications for our inflationary scenario as well as for the cosmological constant problem.

2. Eternal topological inflation

Slow-roll inflation is realized if a scalar potential, $V(\phi)$, is positive in a region where the following conditions are satisfied:

$$\epsilon \equiv \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1\,, \qquad \eta \equiv M_{pl}^2\frac{V''}{V} \ll 1\,. \tag{2.1}$$

Here M_{pl} is the rationalized Planck mass ($(8\pi G)^{-1/2}$) and primes refer to derivatives with respect to the scalar field, which is assumed to be canonically normalized. Satisfying these conditions is not an easy challenge for typical potentials since the inflationary region has to be very flat. Furthermore, after finding such a region we are usually faced with the issue of initial conditions: Why should the field ϕ start in the particular slow-roll domain?

For the simplest chaotic inflation models of the type of $\frac{m^2}{2}\phi^2$ this problem can be easily resolved. In these theories, inflation may start if one has a single domain at a nearly Planckian density, where the field is large and homogeneous on a scale that can be as small as the Planck scale. One can argue that the probability of this event should not be strongly suppressed [23, 25]. Once inflation begins, the universe enters the process of eternal self-reproduction due to quantum fluctuations which unceasingly return some parts of the universe to the inflationary regime. The total volume of space produced by this process is infinitely greater than the total volume of all non-inflationary domains [26, 27].

The problem of initial conditions in the theories where inflation is possible only at the densities much smaller than the Planck density is much more complicated. However, one may still argue that even if the probability of proper initial conditions for inflation is strongly suppressed, the possibility to have eternal inflation infinitely rewards those domains where inflation occurs. One may argue that the problem of initial conditions in the theories where eternal inflation is possible is largely irrelevant, see [27] for a discussion of this issue.

Eternal inflation is not an automatic property of all inflationary models. Many versions of the hybrid inflation scenario, including some of the versions used recently for the implementation of inflation in string theory, do not have this important property. Fortunately, inflation is eternal in all models where it occurs near the flat top of an effective potential. Moreover, it occurs not only due to quantum fluctuations [28], but even at the classical level, due to eternal expansion of topological defects [20, 21].

The essence of this effect is very simple. Suppose that the potential has a saddle point, so that some components of the field have a small tachyonic mass $|m^2| \ll H^2$, where $H^2 = V/(3M_{pl}^2)$, and V is the value of the effective potential at the saddle point. Consider for example a sinusoidal wave of the field ϕ, $\delta\phi \sim \phi_0 \sin kx$ with $k \ll m$. It can be shown that the amplitude of such a wave grows as $e^{|m^2|t/3H} = e^{\eta Ht}$ [25], whereas the distance between the nodes of this wave grows much faster, as e^{Ht}. As a result, at each particular point the field falls down and inflation ends, but the total volume of all points staying close to the saddle point continues growing exponentially, making inflation eternal [20, 21].

3. Racetrack inflation

In this section we exhibit an example of topological inflation within string moduli space, following the KKLT scenario.

3.1 The effective 4D theory

Recall that this scenario builds on the GKP construction [29] (see [30] for earlier discussions), for which type-IIB string theory is compactified on an orientifolded Calabi-Yau manifold in the presence of three-form RR and NS fluxes, as well as D7 branes containing $\mathcal{N} = 1$ supersymmetric gauge field theories within their world-volumes. The background fluxes provide potential energies which can fix the values of the complex dilaton field and of the complex-structure moduli. The resulting effective 4D description of the Kähler moduli is a supergravity with a potential of the no-scale type [31], corresponding to classically flat directions along which supersymmetry generically breaks. KKLT start with Calabi-Yaus having only a single Kähler modulus, and lift this remaining flat direction using nonperturbative effects to induce nontrivial superpotentials for it. After fixing this modulus they introduce anti-D3 branes to lift the minimum of the potential to nonnegative values, leading to a metastable de Sitter space in four dimensions. Alternatively, the same effect as the anti-D3 branes can be obtained by turning on magnetic fluxes on the D7 branes, which give rise to a Fayet-Iliopoulos D-term potential [32]. Other combinations of non-perturbative effects in string theory leading to dS vacua were proposed in [33].

We here follow an identical procedure, based on the dynamics of a single Kähler modulus, T, whose real part measures the volume of the underlying Calabi-Yau space.[2] (For type-IIB theories the imaginary part of T consists of a component of the RR 4-form which couples to 3-branes). Just as for KKLT, all other fields are assumed to have been fixed by

[2]It has been recently argued [35, 36] that nonperturbative superpotentials cannot be generated for a large class of one-modulus Calabi-Yau compactifications, with the authors of these references differing on whether or not the resulting landscape is half-full or half-empty. We do not regard their results as an air-tight no-go theorem for single-modulus vacua until more exhaustive studies of string vacua are performed. For instance, mechanisms like orbifolding and turning on magnetic fluxes on D7 branes could modify the matter spectrum of the $N = 1$ supersymmetric theory within the D7 brane in such a way that nonperturbative superpotentials of the gaugino condensation type could be induced. These issues are now investigated in [34]. In any case, since the single-modulus examples are the simplest scenarios, they can always be seen as some sort of limiting low-energy region — the one for which all but one of the Kähler moduli have been fixed at higher scales — in the many-moduli compactifications that have been found to lead to nonperturbative superpotentials.

JHEP11(2004)063

the background fluxes, and a superpotential, $W(T)$, for T is imagined to be generated, such as through gaugino condensation [3, 37] within the gauge theories on the D7 branes (which depend on T because the volume, Re T, defines the gauge coupling on the D7 branes).

Our treatment differs from KKLT only in the form assumed for the nonperturbative superpotential, which we take to have the modified racetrack form [4, 19]

$$W = W_0 + A\, e^{-aT} + B\, e^{-bT}\,. \tag{3.1}$$

such as would be obtained through gaugino condensation in a theory with a product gauge group. For instance, for an $\mathrm{SU}(N)\times\mathrm{SU}(M)$ group we would have $a = 2\pi/N$ and $b = 2\pi/M$. Because the scale of A and B is set by the cutoff of the effective theory, we expect both to be small when expressed in Planck units [38]. The constant term W_0 represents the effective superpotential as a function of all the fields that have been fixed already, such as the dilaton and complex structure moduli.

This superpotential includes the one used by KKLT as the special case $AB = 0$. It also includes the standard racetrack scenario (when $W_0 = 0$), which was much discussed in order to fix the dilaton field at weak coupling in the heterotic string [4]. Its utility in this regard is seen for large values of N and M, with M close to N, since then the globally-supersymmetric minimum, $W' = 0$, occurs when

$$T = \frac{NM}{M-N} \log\left(-\frac{MB}{NA}\right) \tag{3.2}$$

and so is guaranteed to lie in the region where Re T is large, corresponding to weak coupling.[3] The same proves to be true for minima of the full supergravity potential, for which simple analytic expressions are not available. This superpotential was first adapted to the KKLT scenario in [19].

Following KKLT, the scalar potential we consider is a sum of two parts

$$V = V_F + \delta V\,. \tag{3.3}$$

The first term comes from the standard $\mathcal{N} = 1$ supergravity formula for the F-term potential, which in Planck units [39] reads

$$V_F = e^K \left(\sum_{i,j} K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right), \tag{3.4}$$

where i, j runs over all moduli fields, K is the Kähler potential for T, $K^{i\bar{j}}$ is the inverse of $\partial_i \partial_{\bar{j}} K$, and $D_i W = \partial_i W + (\partial_i K) W$. For the Kähler potential, K, we take the weak-coupling result obtained from Calabi-Yau compactifications [40], namely

$$K = -3 \log(T + T^*)\,. \tag{3.5}$$

We neglect the various possible perturbative and nonperturbative corrections to this form which might arise.

[3]This is the main idea behind the standard racetrack scenarios, originally proposed for the heterotic string in order to get minima with small gauge coupling constant. Notice that the large value of T corresponding to $W' = 0$ is independent of the value of W_0, which was not introduced in the original racetrack models but plays an important role here.

JHEP11(2004)063

The nonsupersymmetric potential, δV, is that part of the potential which is induced by the tension of the anti-D3 branes.[4] The introduction of the anti-brane does not introduce extra translational moduli because its position is fixed by the fluxes [41], so it just contributes to the energy density of the system. This contribution is positive definite and depends on a negative power of the Calabi-Yau volume, $X = \mathrm{Re}\, T$, as follows [41]:

$$\delta V = \frac{E}{X^\alpha}\,, \tag{3.6}$$

where the coefficient E is a function of the tension of the brane T_3 and of the warp factor. The exponent α is either $\alpha = 2$ if the anti-D3 branes are sitting at the end of the Calabi-Yau throat, or in the case of magnetic field fluxes [32], if the D7 branes are located at the tip of the throat. Otherwise $\alpha = 3$ corresponding to the unwarped region (in the anti-D3 brane case the warped region is energetically preferred and we will usually take $\alpha = 2$). There is clearly considerable model dependence in this scenario. It depends on the number of Kähler moduli of the original Calabi-Yau manifold, on what kind of nonperturbative superpotential can be induced for them (if any), on the Kähler function, the power α, and so on.

3.2 The scalar potential

We now explore the shape of the scalar potential, to identify potential areas for slow-roll inflation. To this end we write the field T in terms of its real and imaginary parts:

$$T \equiv X + iY\,. \tag{3.7}$$

Notice that, to the order that we are working, the Kähler potential depends only on X and not on Y. For fields rolling slowly in the Y direction this feature helps address the η problem of F-term inflation.

Using (3.4) and (3.5) the scalar potential turns out to be

$$V_F = \frac{1}{8X^3}\left\{\frac{1}{3}|2XW' - 3W|^2 - 3|W|^2\right\}, \tag{3.8}$$

where $'$ denotes derivatives with respect to T. The supersymmetric configurations are given by the solutions to

$$2XW' - 3W = 0\,. \tag{3.9}$$

Using only V_F, the values of the potential at these configurations are either negative or zero, corresponding to anti-de Sitter or Minkowski vacua. Substituting the explicit form of the superpotential and adding the SUSY-breaking term we find that the scalar potential becomes

$$V = \frac{E}{X^\alpha} + \frac{e^{-aX}}{6X^2}\left[aA^2\,(aX+3)\,e^{-aX} + 3W_0 aA\cos(aY)\right] +$$

[4] In [32] the anti-D3 brane was substituted by magnetic fluxes on D7 branes. The potential generated is identical to the one induced by the anti-D3 brane, with the advantage of having the interpretation of a supersymmetric Fayet-Iliopoulos D-term.

$$+\frac{e^{-bX}}{6X^2}\left[bB^2\,(bX+3)\,e^{-bX}+3W_0 bB\cos(bY)\right]+$$
$$+\frac{e^{-(a+b)X}}{6X^2}\left[AB\,(2abX+3a+3b)\cos((a-b)Y)\right]. \tag{3.10}$$

This potential has several de Sitter (or anti-de Sitter) minima, depending on the values of the parameters A, a, B, b, W_0, E. In general it has a very rich structure, due in part to the competition of the different periodicities of the Y-dependent terms. In particular, $a-b$ can be very small, as in standard racetrack models, since we can choose $a = 2\pi/M$, $b = 2\pi/N$ with $N \sim M$ and both large integers. Notice that in the limit $(a-b) \to 0$ and $W_0 \to 0$, the Y direction becomes exactly flat. We can then tune these parameters (and AB) in order to obtain flat regions suitable for inflation.

From the above we expect extrema situated at large $X = \mathrm{Re}\, T$ given a discrete fine tuning of M and N. This is independent of the value of W_0 and in particular occurs for $W_0 = 0$. This behaviour is different from the original KKLT scenario, which does not have any minima when $W_0 = 0$. For $W_0 \neq 0$ many new local minima appear due to the periodicity of the terms proportional to W_0 in the scalar potential. For the minima of interest we use the freedom to choose the value of E, as in KKLT, to tune the global minimum of the potential to the present-day vacuum energy.

Furthermore, the shape of the potential is very sensitive to the values of the parameters. We find that the potential has a maximum in the Y- direction if the following conditions are satisfied: $A + B < 0$, $W_0 < 0$, $a < b$. Otherwise the point $Y = 0$ would correspond to a minimum. With these condtions satisfied and for a fixed value of the other parameters, there is a critical value of W_0 beyond which the point $Y = 0$ is also a maximum in the X-direction and therefore the field runs away to the uncompactified limit $X \to \infty$. But for W_0 smaller than the critical value, the point $Y = 0$ is a minimum in the X-direction and therefore a saddle point. This is the interesting range to look for slow-roll inflation.

Figures 1 and 2 illustrate a region of the scalar potential for which inflation is possible. The values of the parameters which are used to obtain this potential are:

$$A=\frac{1}{50}\,,\qquad B=-\frac{35}{1000}\,,\qquad a=\frac{2\pi}{100}\,,\qquad b=\frac{2\pi}{90}\,,\qquad W_0=-\frac{1}{25000}\,. \tag{3.11}$$

With these values the two minima seen in the figure occur for field values

$$X_{\mathrm{min}} = 96.130\,,\qquad Y_{\mathrm{min}} = \pm 22.146\,, \tag{3.12}$$

and the inflationary saddle point is at

$$X_{\mathrm{saddle}} = 123.22\,,\qquad Y_{\mathrm{saddle}} = 0\,. \tag{3.13}$$

The value of E is fixed by demanding that the value of the potential at this minimum be at zero, which turns out to require $E = 4.14668\times 10^{-12}$. We find this to be a reasonable value given that E is typically suppressed by the warp factor of the metric at the position of the anti-brane.

JHEP11(2004)063

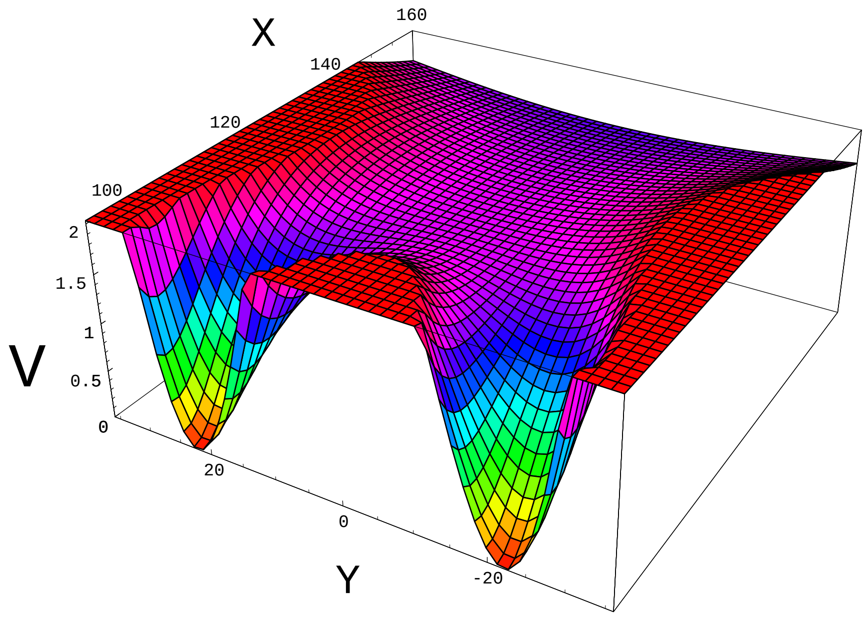

Figure 1: Plot for a racetrack type potential (rescaled by 10^{16}). Inflation begins in a vicinity of the saddle point at $X_{\rm saddle} = 123.22$, $Y_{\rm saddle} = 0$. Units are $M_p = 1$.

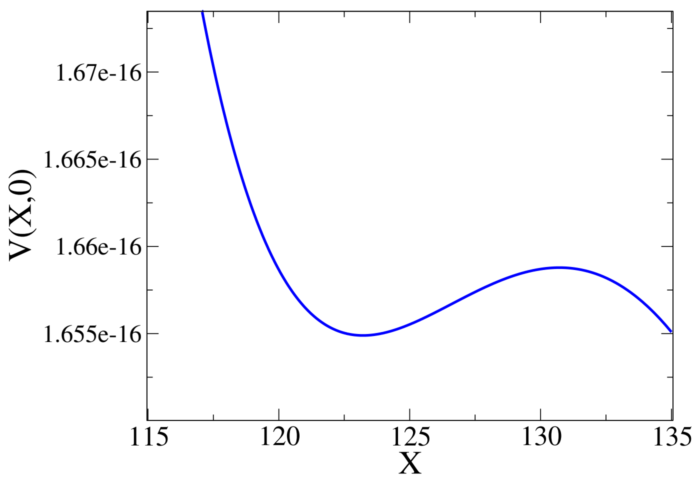

Figure 2: Plot of $V(X, 0)$ versus X in the vicinity of the saddle point $X_{\rm saddle} = 123.22$, $Y_{\rm saddle} = 0$.

It is crucial that this model contains two degenerate minima[5] since this guarantees the existence of causally disconnected regions of space which are in different vacua. These regions necessarily have a domain wall between them where the field is near the saddle point and thus eternal inflation is taking place, provided that the slow roll conditions (2.1) are satisfied there. It is then inevitable to have regions close to the saddle in which inflation occurs, with a sufficiently large duration to explain our flat and homogeneous universe.

[5]Note that the potential is periodic with period 900, i.e. there is a set of two degenerate minima at every $Y = 900\ n$ where $n = 0, 1, 2, \ldots$. etc.

3.3 Scaling properties of the model

It is easy to see from the potential (3.10) that we can obtain models with rescaled values of the critical points for other choices of parameters, having the same features of inflation. This can be done simply by rescaling their values in the following way,

$$a \to \frac{a}{\lambda}, \qquad b \to \frac{b}{\lambda}, \qquad E \to \lambda^2 E, \tag{3.14}$$

and also

$$A \to \lambda^{3/2} A, \qquad B \to \lambda^{3/2} B, \qquad W_0 \to \lambda^{3/2} W_0, \tag{3.15}$$

Under all these rescalings the potential does not change under condition that the fields also rescale

$$X \to \lambda X, \qquad Y \to \lambda Y, \tag{3.16}$$

in which case the location of the extrema also rescale. One can verify that the values of the slow-roll parameters ϵ and η do not change and also the amplitude of the density perturbations $\delta\rho/\rho$ remains the same. It is important to take into account that the kinetic term in this model is invariant under the rescaling, which is not the case for canonically normalized fields.

Another property of this model is given by the following rescalings

$$a \to \frac{a}{\mu}, \qquad b \to \frac{b}{\mu}, \qquad E \to \frac{E}{\mu}, \tag{3.17}$$

The potential and the fields also rescale

$$V \to \mu^{-3} V, \qquad X \to \mu X, \qquad Y \to \mu Y, \tag{3.18}$$

Under these rescalings the values of the slow-roll parameters ϵ and η do not change however, the amplitude of the density perturbations $\delta\rho/\rho$ scales as $\mu^{-3/2}$.

These two types of rescalings allow to generate many other models from the known ones, in particular, change the positions of the minima or, if one is interested in eternal inflation, one can easily change $\delta\rho/\rho$ keeping the potential flat.

3.4 Slow-roll inflation

We now display the slow-roll inflation, by examining field motion near the saddle point which occurs between the two minima identified above. Near the saddle point $X_{\text{saddle}} = 123.22$, $Y_{\text{saddle}} = 0$, the potential takes the value $V_{\text{saddle}} = 1.655 \times 10^{-16}$. At this saddle point the potential has a maximum in the Y direction and a minimum in the X direction, so the initial motion of a slowly-rolling scalar field is in the Y direction.

We compute the slow-roll parameters near the top of the saddle, keeping in mind the fact that Y does not have a canonical kinetic term. Given the Kähler potential we find that the kinetic term for X and Y is

$$\mathcal{L}_{\text{kin}} = \frac{3M_p^2}{4X^2} \left(\partial_\mu X \partial^\mu X + \partial_\mu Y \partial^\mu Y\right) \tag{3.19}$$

$10^3\,A$	$-10^3\,B$	$-\frac{10^{-3}}{W_{\min}}$	$-\frac{10^{-3}}{W_{\max}}$
20	35	24.998	25.000
20	34	20.389	20.400
10	16	26.766	26.780
5	7.5	34.496	34.520
5	7	21.800	21.824
$3\frac{1}{3}$	$4\frac{1}{3}$	20.280	20.304
$2\frac{6}{7}$	$3\frac{3}{7}$	14.420	14.448
$2\frac{6}{7}$	$3\frac{1}{7}$	8.6884	8.708

Table 1: A range of parameters chosen to satisfy $-0.05 < \eta < 0$, close to the parameter region (3.11).

$-B$	$-1/W_{\min}$	$-1/W_{\max}$
3.5	90685.8	90667.55
3.3	57273.9	57286.2
3.0	27371.1	27377.8
2.8	16111.8	16116.17
2.4	5009.38	5011.125
2.0	1301.6	1302.23
1.6	267.445	267.651

Table 2: Another such range of parameters, with $a = \pi/5$, $b = 2\pi/9$ and $A = 1$.

and so the correctly normalized η parameter is given by the expression $\eta = 2X^2V''/3V$, with X evaluated at the saddle point. We find in this way the slow-roll parameters

$$\epsilon_{\text{saddle}} = 0\,, \qquad \eta_{\text{saddle}} = -0.006097\,, \tag{3.20}$$

given the values of the parameters taken above. The small size of η is very encouraging given the reasonable range of parameters chosen. Recall from section 2 that the slow-roll condition $\eta \ll 1$ being satisfied at the saddle point implies automatically that we have an inflationary regime in its vicinity, and that inflation starting here is an example of eternal topological inflation.

How fine-tuned is this parameter choice we have found which accomplishes inflation? Its success relies on the second derivative, $\partial^2 V/\partial Y^2$, passing through a zero very close to the saddle point (where $\partial V/\partial Y$ vanishes). Our ability to accomplish this relies on the freedom to adjust W_0, a parameter which is not present in a pure racetrack model. To study the range of parameters which produce inflation we explored the vicinity of the parameters (3.11) which produce an inflationary solution. We were able to preserve the condition $-0.05 < \eta < 0$ by varying the parameters W_0 and B, while at all times adjusting E to keep the potential's minimum at zero. For many choices of B, sufficiently small η was obtained for $W_{\min} < W_0 < W_{\max}$, given in table 1. The same is done in table 2 for a different inflationary region of parameter space. We see from these tables that the success of slow-roll inflation typically requires a fine tuning of parameters at the level of 1 part in 1000. Parameter values in table 1 were chosen so as to respect the COBE normalization (see below). This constraint was not imposed for the values chosen in table 2, but this can always be compensated using the rescaling (3.17).

To compute observable quantities for the CMB we numerically evolve the scalar field starting close to the saddle point, and let the fields evolve according to the cosmological evolution equations for non-canonically normalized scalar fields [42, 15, 43, 44]:

$$\ddot{\varphi}^i + 3H\dot{\varphi}^i + \Gamma^i_{jk}\,\dot{\varphi}^j\,\dot{\varphi}^k + g^{ij}\,\frac{\partial V}{\partial \varphi^j} = 0\,,$$

JHEP11(2004)063

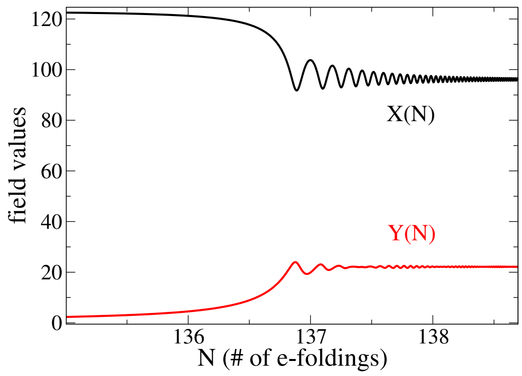

Figure 3: Evolution of X (upper curve) and Y (lower curve) from their initial values near the saddle point, to one of the degenerate minima of the potential.

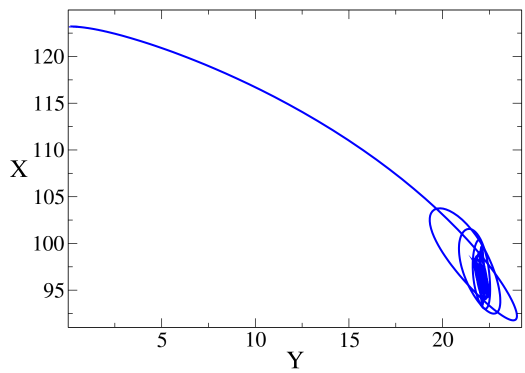

Figure 4: Path of inflaton trajectory in X-Y field space.

JHEP11(2004)063

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\frac{1}{2}g_{ij}\dot{\varphi}^i\dot{\varphi}^j + V\right), \tag{3.21}$$

where φ_i represent the scalar fields ($\mathrm{Re}\,T \equiv X$ and $\mathrm{Im}\,T \equiv Y$ in our case), a is the scale factor (not to be confused with the exponent in the superpotential), and Γ^i_{jk} are the target space Christoffel symbols using the metric g_{ij} for the set of real scalar fields φ^i such that $\frac{\partial^2 K}{\partial\Phi^I\partial\Phi^{J*}}\partial\Phi^I\partial\Phi^{*J} = \frac{1}{2}g_{ij}\partial\varphi^i\partial\varphi^j$.

For numerical purposes it is more convenient to write the evolution of the fields as a function of the number N of e-foldings rather than time. Using

$$a(t) = e^N\,, \qquad \frac{d}{dt} = H\frac{d}{dN}\,, \tag{3.22}$$

we avoid having to solve for the scale factor, instead directly obtaining $X(N)$ and $Y(N)$. The equations of motion are

$$\begin{aligned} X'' &= -\left(1 - \frac{X'^2 + Y'^2}{4X^2}\right)\left(3X' + 2X^2\frac{V_{,X}}{V}\right) + \frac{X'^2 - Y'^2}{X} \\ Y'' &= -\left(1 - \frac{X'^2 + Y'^2}{4X^2}\right)\left(3Y' + 2X^2\frac{V_{,Y}}{V}\right) + 2\frac{X'Y'}{X} \end{aligned} \tag{3.23}$$

where $'$ denotes d/dN. The results of the numerical evolution are shown in figure 3 and 4, using the parameters (3.11) and the initial conditions $X = X_{\rm saddle}$ (from (3.13)) and $Y = 0.1$. Figure 3 shows that this choice gives approximately 137 e-foldings of inflation before the fields start oscillating around one of the local minima. Figure 4 illustrates that the inflaton is primarily Y at the very beginning of inflation, as must be the case since Y is the unstable direction at the saddle point. Starting at $Y = 0.2$ would give 63 rather than 137 e-foldings of inflation. However, as it has already been explained, tuning of initial conditions is not a concern within the present model, where inflation is topological and eternal; all possible initial conditions will be present in the global spacetime.

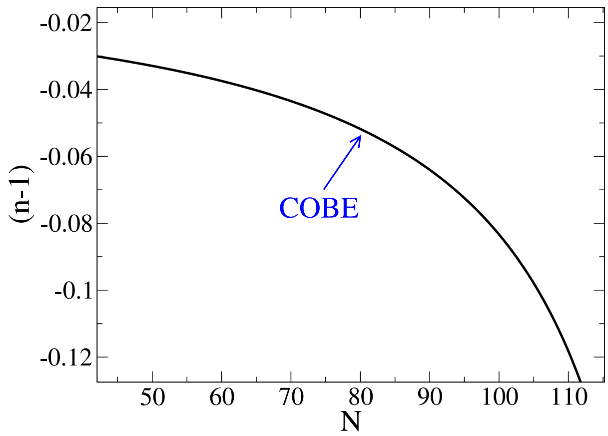

Figure 5: Deviation of the scalar power spectral index from 1 $(n-1)$ versus N (equivalent to $n-1$ versus $\ln k$) for the typical racetrack model.

3.5 Experimental constraints and signatures

Let us now consider the experimental constraints on and predictions of the racetrack inflation model. First, we must satisfy the COBE normalization on the power spectrum of scalar density perturbations, $\sqrt{P(k)} = 2 \times 10^{-5}$, at the scale $k_0 \sim 10^3$ Mpc, or equivalently at the value of N which is approximately 60 e-foldings before the end of inflation. We ignore isocurvature fluctuations (arising from fluctuations of the fields orthogonal to the inflaton path shown in figure 4) since there is always a hierarchy between the second derivative of the potential along the path relative to that along the orthogonal direction; then the magnitude of the scalar power spectrum can be approximated by either

$$P_1(k) = \frac{1}{50\pi^2}\frac{H^4}{\mathcal{L}_{\rm kin}} \qquad \text{or} \qquad P_2(k) = \frac{1}{150\pi^2}\frac{V}{\epsilon} \tag{3.24}$$

where the generalization of the slow-roll parameter ϵ in the two-field case is given by

$$\epsilon = M_p^2 \frac{(V_{,X}\dot{X} + V_{,Y}\dot{Y})^2}{4\,\mathcal{L}_{\rm kin}\, V^2}\,. \tag{3.25}$$

We have numerically checked that the two formulas give consistent results during the slow-roll period, and that the COBE normalization is satisfied for the parameters given in (3.11).

The spectral index is defined to be

$$n_s = 1 + \frac{d\ln P(k)}{d\ln k} \cong 1 + \frac{d\ln P(N)}{dN} \tag{3.26}$$

where the latter approximation follows from the fact that $k = aH \cong He^N$ at horizon crossing, so $d\ln k \cong dN$. In figure 5 we plot $n_s - 1$ versus N, showing that $n_s \approx 0.95$ in the COBE region of the spectrum. This red value is typical for inflationary potentials with negative curvature. From the slope we see that $dn/d\ln k \cong -0.001$, so that the running of the index is negligible relative to the current experimental sensitivity, also typical for models in which the flatness of the potential is not punctuated by any special features.

JHEP11(2004)063

The value of the spectral index $n_s \approx 0.95$ and the smallness of the running index appear to be pretty stable with respect to various modifications of the model; we were unable to alter these results by changing various parameters. This makes racetrack inflation testable; at present, the best constraint on n_s is $n_s = 0.98 \pm 0.02$ [47], which is compatible with our results. Future experiments will be able either confirm our model or rule it out; in particular, Planck satellite will measure n_s with accuracy better than 0.01.

It is interesting that experimental tests will be able to discriminate for or against the model in the not-too-distant future. The presence of a tensor contribution will not provide any test in the example we have considered since the scale of inflation is $V^{1/4} \sim 10^{14}$ GeV, far below the 3×10^{16} GeV threshold needed for producing observable gravity waves.

In order to formulate the theory of reheating in our scenario one needs to find a proper way to incorporate the standard model matter fields in our model. In the KKLT scenario there are two possible places where the standard model particles can live: on (anti) D3 branes at the end of the throat [45] or on the wrapped D7 branes, after some twisting and/or turning-on of magnetic fields. If the standard model fields live on a D7 brane, the axion couples to the vector fields like $Y\, F_{\mu\nu}\tilde{F}_{\mu\nu}$ and the volume modulus as $XF_{\mu\nu}F_{\mu\nu}$. To study the reheating one has to find the decay rate of the X and Y fields to the vector particles and the corresponding reheating temperature. A preliminary investigation of this question along the lines of ref. [46] shows that reheating in this scenario can be rather efficient.[6] Explicit models of this type are yet to be constructed. If the standard model is on D3 branes, we would have to consider other couplings of the T-field to matter fields. We hope to return to the theory of reheating in the racetrack inflation scenario in a separate publication.

4. The overshooting problem and initial conditions for the racetrack inflation

Even though we already discussed advantages of eternal topological inflation from the point of view of the problem of initial conditions, we will revisit this issue here again, emphasizing specific features of eternal inflation in the context of string theory.

There is a well-known problem related to initial conditions in string cosmology [48]. This problem, in application to the KKLT-based models, can be formulated as follows. Even though there is a KKLT minimum of the potential with respect to the dilaton field and the volume modulus X, this minimum is separated from the global Minkowski minimum at $X \to \infty$ by a relatively small barrier. The height of the barrier depends on the parameters of the model, but in the simplest models considered in the literature it is 10 to 20 orders of magnitude smaller than the Planck density. In particular, for the parameters of our model, the height of the barrier is 2×10^{-16} in Planckian units, see figure 1. Generically, one could expect that soon after the big bang, the energy density of all fields, including the field X, was many orders of magnitude greater than the height

[6] This situation will be the same in heterotic string realizations of our scenario (with the role of the T field taken by the dilaton S) in which the dilaton field couples universally to hidden and observable gauge fields.

of the barrier. If, for example, the field X initially was very small, with energy density much larger than 2×10^{-16}, then it would fall down along the exponentially steep potential, easily overshoot the KKLT minimum, roll over the barrier, and continue rapidly rolling towards $X \to \infty$. This would correspond to a rapid decompactification of the 4D space.

There are several ways to avoid this problem. One may try, for example, to evaluate the possibility that instead of being born in a state with small $X = \mathrm{Re}\, T$, the universe was created "from nothing" in a state corresponding to the inflationary saddle point of the potential $V(T)$. This would resolve the problem of overshooting. However, at the first glance, instead of solving the problem of initial conditions, this only leads to its even sharper formulation.

Indeed, according to [49, 50], the probability of quantum creation of a closed universe in a state corresponding to an extremum of its effective potential is given by $P \sim \exp\left(-\frac{24\pi^2}{V(T)}\right)$, where the energy density is expressed in units of the Planck density. The same expression appears for quantum creation of an open universe [51]. This leads to an alternative formulation of the problem of initial conditions in our scenario: The probability that a closed (or open) universe is created from "nothing" at the saddle point with $V \sim 2 \times 10^{-16}$ is exponentially suppressed by the factor of $P \sim \exp\left(-10^{18}\right)$. This result, taken at its face value, may look pretty upsetting. (The simplest models of chaotic inflation, which can begin at $V(\phi) = O(1)$, do not suffer from this problem [25].)

In section 2 we outlined a possible resolution of this problem: Even if the probability of proper initial conditions for eternal topological inflation is extremely small, the parts of the universe where these conditions are satisfied enter the regime of eternal inflation, producing infinite amount of homogeneous space where life of our type is possible. Thus, even if the fraction of the universes (or of the parts of our universe) with inflationary initial conditions is exponentially suppressed, one may argue that eventually most of the observers will live in the parts of the universe produced by eternal topological inflation.

Here we would like to strengthen this argument even further, by finding the conditions which may allow us to remove the exponential suppression of the probability of initial conditions for the low energy scale inflation. In order to do it, let us identify the root of the problem of the exponential suppression: At the classical level, the minimal size of a closed de Sitter space is H^{-1}. Quantum creation of a closed inflationary universe is described by the tunneling from the state with the scale factor $a = 0$ (no universe) to the state where the size of the universe becomes equal to its minimal value $a = H^{-1}(T)$. Exponential suppression appears because of the large absolute value of the euclidean action on the tunneling trajectory [49, 50]. One could expect that for an open universe there is no need for tunneling because there is no barrier for the classical evolution of an open universe from $a = 0$ to larger a. However, an instant creation of an *infinite* homogeneous open universe is problematic (homogeneity and horizon problems). The only known way to describe it is to use a different analytic continuation of the same instantons that were used for the description of quantum creation of a closed universe [52], which again leads to the exponential suppression of the probability [51].

Fortunately, there is a simple way to overcome this problem [53], which is directly related to the standard Kaluza-Klein picture of compactification in string theory. In this picture, it is natural to assume that *all* spatial dimensions enter the theory democratically, i.e. all of them are compact, not necessarily because of the curvature of space (as in the closed universe case), but because of its nontrivial topology. Then inflation makes the size of 3 of these dimensions exponentially large, whereas the size of 6 other dimensions remains fixed, e.g., by the KKLT mechanism.

For example, one may consider a toroidal compactification of a flat universe, or compactification of an open universe. This is a completely legitimate possibility, which was investigated by many authors; see e.g. [54]–[57]. It does not contradict any observational data if inflation is long enough to make the size of the universe much greater than 10^{10} light years.

An important feature of a topologically nontrivial compact flat or open dS space is that (ignoring the Casimir effect which is suppressed by supersymmetry) its classical evolution can continuously proceed directly from the state with $a(t) \to 0$, without any need of tunneling, unlike in the closed universe case [58]. As a result, *there is no exponential suppression of the probability of quantum creation of a compact flat or open inflationary universe with* $V(T) \ll 1$ [53]. This observation removes the main objection against the possibility of the low energy scale inflation starting at an extremum of the effective potential.

Moreover, in an eternally inflating universe consisting of many de Sitter parts corresponding to the string theory landscape, the whole issue of initial conditions should be formulated in a different way [27, 59]. In such a universe, inflation is always eternal because of the incomplete decay of metastable dS space, as in old inflation. Therefore evolution of such a universe will produce infinitely large number of exponentially large parts of the universe with different properties [27, 62]. Even if in many of such parts space will become 10D after the scalar field T overshoots the KKLT barrier, this will be completely irrelevant for the evolution of other parts of the universe where space remains 4D. In some parts of 4D space there was no inflation, and adiabatic perturbations with a flat spectrum could not be produced. Even though such parts initially may be quite abundant, later on they do not experience an additional inflationary growth of their volume. Observations tell us that we do not live in one of such parts. Meanwhile inflationary trajectories starting at the saddle point describe eternally inflating parts of the universe which produce an indefinitely large volume of homogeneous 4D space where observers of our type can live. This observation goes long way towards resolving the problem of initial conditions in our scenario.

5. Irrational racetrack inflation and the cosmological constant

In order to achieve a successful cosmological scenario, one needs to make at least two fine-tunings. First of all, we must fine-tune the uplifting of the AdS potential in the KKLT scenario to obtain the present value of the cosmological constant $\Lambda \sim 10^{-120}$ in units of Planck density. Then we must fine-tune the parameters of our model in order to achieve a slow-roll inflation with the amplitude of density perturbations $\delta\rho/\rho \sim 10^{-5}$.

JHEP11(2004)063

One way to approach the fine-tuning problem is to say that the smallness of the cosmological constant, as well as the stage of inflation making our universe large and producing small density perturbations are necessary for the existence of life as we know it. This argument may make sense in the context of the eternal inflation scenario, but only if there are sufficiently many vacuum states with different values of the cosmological constant, and one can roll down to these states along many different inflationary trajectories.

In this respect, the possibility that the eternally inflating universe becomes divided into many exponentially large regions with different properties [25, 60] and the related idea of the string theory landscape describing enormously large number of different vacua may be very helpful [61, 62, 35]. However, even in this case one must check whether the set of parameters which are possible in string theory is dense enough to describe theories with $\Lambda \sim 10^{-120}$ and $\delta\rho/\rho \sim 10^{-5}$.

One way towards making the choice of various vacua infinitely rich is related to the irrational axion scenario [63]. In our previous discussion of the effective potential of our model, eq. (3.10), we assumed that $a = 2\pi/M$ and $b = 2\pi/N$, where N and M are integers. In this case the potential is periodic in Y, with a period MN, or less, $MN/(M-N)$ if $M/(M-N)$ and $N/(M-N)$ are integers. For example, if one takes $M = 100$ and $N = 90$, the potential will have identical inflationary saddle points for all $Y = n(MN/(M-N)) = 900n$, where n is an arbitrary integer.

However, if one assumes that instead of being an integer, at least one of the numbers M, N is irrational, then in the infinitely large interval of all possible values of the axion field Y values one would always find vacua with all values of Λ varying on a very large scale comparable with the height of the potential barrier in the KKLT potential. Similarly, if we, for example, start with an irrational number M very close to 1000, then at $Y = 0$ we will have the same scenario as in the model described above, but at large Y we will have all kinds of extrema and saddle points, leading to various stages of inflation with different amplitude of density perturbations.

To illustrate this idea, we give here the contour plots of two subsequent saddle points which appear in our theory under a very mild modification: We replace $M = 100$ by $M = 100 + \pi/100$. It immediately produces an infinitely large variety of different shape saddle points and minima of different depth; we show two different saddle points separated by $Y \approx 900$ in figure 6. The shape and the depth of the minima of these potentials can be changed in an almost continuous way. This reflects the situation with irrational winding line on the torus for incommensurate frequencies: the system will never return to its original position, it will cover the torus densely, coming arbitrary close to every point.

The main question is whether one can find a version of string theory with the superpotential involving irrational M. Whereas no explicit examples of such models are known at present, the idea is so attractive that we allow ourselves to speculate about its possible realizations. One can try to use the setting of [64] where the non-commutative solitons on orbifolds are studied and some relation to irrational axions is pointed out. One can also look for the special contribution to the Chern-Simons term on D7 due to the 2-form fluxes. The standard aFF^* axion coupling originates from the term $\int_{M^9} d[Ce^{\mathcal{F}}]$ where one finds $d[Ce^{\mathcal{F}}] \sim da \wedge F \wedge F \wedge J \wedge J$ and J is the Kähler form and $da \wedge J \wedge J$ comes

JHEP11(2004)063

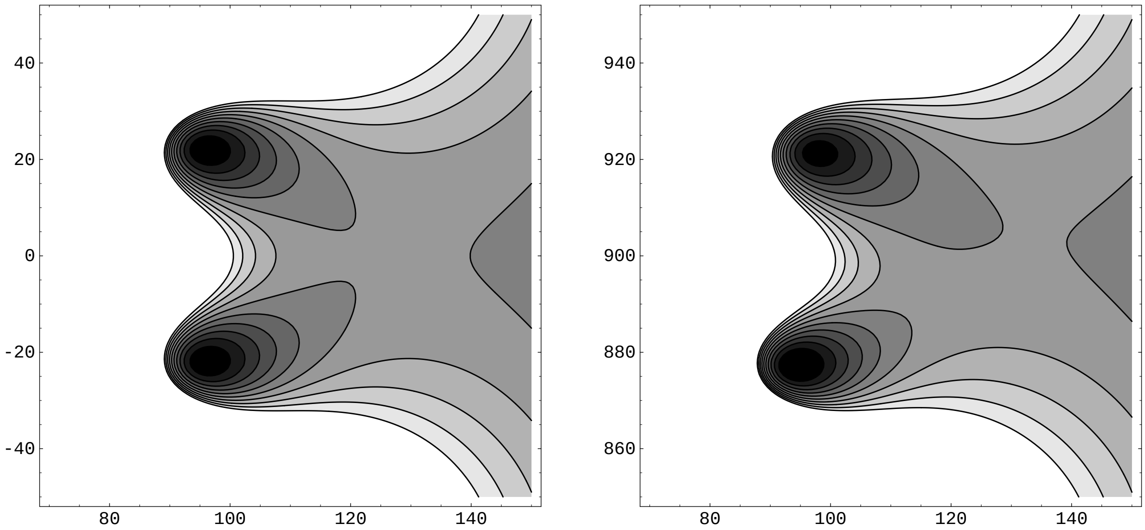

Figure 6: Contour plot of the effective potential in the vicinity of the saddle point at $Y = 0$ (left panel) and at $Y \sim 900$ (right panel).

from the self-dual five-form. Perhaps it is not impossible to find in addition to this term a contribution where one or both of the Kähler forms are replaced by the 2-form fluxes $\mathcal{F} = F - B$. Here the 2-form B is not quantized and is related to the non-commutativity parameter in the internal space. This may lead to some new coupling of the form $qaFF^*$ where q is irrational as proposed in [63].

6. Conclusions

We present what we believe is the simplest model for inflation in a string-theoretic scenario. It requires a certain amount of fine-tuning of the parameters of the non-perturbative racetrack superpotential. Inflation only occurs due to the existence of a nontrivial potential lifting the complex Kähler structure modulus whose real part is the overall size of the compact space and the imaginary part coming from the type-IIB four-form is the inflaton field. The periodicity properties in this direction make the scalar potential very rich and we find explicit configurations realizing the slow-roll/topological inflation setting that give rise to eternal inflation and density perturbations in our patch consistent with the COBE normalization and spectral index inside (but close to the edge of) the current observational bounds. We find this to be very encouraging.

It is appropriate to compare our scenario with the other direction for obtaining inflation in string theory, namely the brane/brane [6, 10] or brane/antibrane [7, 8] inflationary scenarios in which the inflaton field corresponds to the separation of the branes ψ and inflation ends by the appearance of an open string tachyon in a stringy realization of hybrid inflation [7, 10]. Clearly our racetrack scenario is much simpler since there is no need to introduce the brane configurations with their separation field ψ to obtain inflation. The analysis of [13] pointed out the difficulty in obtaining brane inflation once a mechanism for moduli fixing is included. Various extensions and improvements of the basic scenario

JHEP11(2004)063

were considered in [13, 15, 16], they typically require a fine-tuning. At present only D3/D7 brane inflation model compactified on $K3 \times \frac{T_2}{Z_2}$ with volume stabilization does not seem to require a fine-tuning [65].

In addition, in brane inflation scenario the tachyon potential can give rise to topological defects such as cosmic strings [7, 13, 66] which may have important observational implications [66, 67]. Furthermore, having an implementation of inflation in string theory, independent of brane inflation, diminishes in some sense the possible observational relevance of cosmic strings as being a generic implication of the brane inflation scenario. In racetrack inflation, cosmic strings need not be generated after inflation.

On the other hand there is no inconsistency to combine both scenarios. For instance, in realistic generalizations of the KKLT scenario including standard model branes [45], the throat where the standard model lives should be significantly warped, whereas brane/antibrane inflation required a mild warping in the inflation throat in order to be consistent with the COBE normalization [15]. Also the fine tuning involved suggested two or more stages of inflation with some 20-30 e-foldings at a high scale and the rest at a small scale, probably close to 1 TeV. This, besides having interesting observational implications, may also help to the solution of the cosmological moduli problem [68] which usually needs a late period of inflation. We could then foresee a scenario in which racetrack inflation is at work at high scales and the low-scale inflation is provided by brane/antibrane collision in the standard model throat. See [69] for a previous discussion of low-scale inflation.

There are many avenues along which to generalize our approach. Modular inflation could also be obtained by using more general modular superpotentials than those of the racetrack type considered here. Also, potentials for many moduli fields are only starting to be explored and some of them have the advantage that a nonvanishing superpotential has been obtained [35]. It would also be interesting to investigate if the irrational cases proposed here are actually obtainable from string theory. We hope to report on some of these issues in the future.

Acknowledgments

We would like to thank R. Brustein, E. Copeland, S.P. de Alwis, G. Dvali, A. Font, S. Gratton, A. Gruzinov, A. Iglesias, X. de la Ossa, L. Ibáñez, S. Kachru, N. Kaloper, G. Ross, S. Sarkar, R. Scoccimarro, E. Silverstein, H. Tye and A. Vilenkin for useful conversations. We also thank the organizers of the 'New Horizons in String Cosmology' workshop at the Banff International Research Station for providing the perfect environment for completing this work and for contemplating the landscape. C.B. and J.C. are funded by grants from NSERC (Canada), NATEQ (Québec) and McGill University. The work of M.G.-R. is supported by D.O.E. under grant DE-FG02-92ER40706. J.J.B-P. is supported by the James Arthur Fellowship at NYU. C.E. is partially funded by EPSRC. The work of R.K. and A.L. is supported by the NSF (US) grant 0244728. F.Q. is partially funded by PPARC and a Royal Society Wolfson award.

JHEP11(2004)063

References

[1] P. Binetruy and M.K. Gaillard, *Candidates for the inflaton field in superstring models*, *Phys. Rev.* **D 34** (1986) 3069.

[2] T. Banks, M. Berkooz, S.H. Shenker, G.W. Moore and P.J. Steinhardt, *Modular cosmology*, *Phys. Rev.* **D 52** (1995) 3548 [hep-th/9503114].

[3] J.P. Derendinger, L.E. Ibáñez and H.P. Nilles, *On the low-energy $D = 4$, $N = 1$ supergravity theory extracted from the $D = 10$, $N = 1$ superstring*, *Phys. Lett.* **B 155** (1985) 65;
M. Dine, R. Rohm, N. Seiberg and E. Witten, *Gluino condensation in superstring models*, *Phys. Lett.* **B 156** (1985) 55.

[4] N.V. Krasnikov, *On supersymmetry breaking in superstring theories*, *Phys. Lett.* **B 193** (1987) 37;
L.J. Dixon, *Supersymmetry breaking in string theory*, in *The Rice meeting*, B. Bonner and H. Miettinen eds., World Scientific, Singapore 1990;
T.R. Taylor, *Dilaton, gaugino condensation and supersymmetry breaking*, *Phys. Lett.* **B 252** (1990) 59;
B. de Carlos, J.A. Casas and C. Muñoz, *Supersymmetry breaking and determination of the unification gauge coupling constant in string theories*, *Nucl. Phys.* **B 399** (1993) 623 [hep-th/9204012].

[5] A. Font, L. E. Ibáñez, D. Lüst and F. Quevedo, *Supersymmetry breaking from duality invariant gaugino condensation*, *Phys. Lett.* **B 245** (1990) 401;
S. Ferrara, N. Magnoli, T. R. Taylor and G. Veneziano, *Duality and supersymmetry breaking in string theory*, *Phys. Lett.* **B 245** (1990) 409;
M. Cvetič, A. Font, L.E. Ibáñez, D. Lüst and F. Quevedo, *Target space duality, supersymmetry breaking and the stability of classical string vacua*, *Nucl. Phys.* **B 361** (1991) 194.

[6] G.R. Dvali and S.H.H. Tye, *Brane inflation*, *Phys. Lett.* **B 450** (1999) 72 [hep-ph/9812483].

[7] C.P. Burgess et al., *The inflationary brane-antibrane universe*, *J. High Energy Phys.* **07** (2001) 047 [hep-th/0105204].

[8] G.R. Dvali, Q. Shafi and S. Solganik, *D-brane inflation*, hep-th/0105203.

[9] C.P. Burgess, P. Martineau, F. Quevedo, G. Rajesh and R.J. Zhang, *Brane antibrane inflation in orbifold and orientifold models*, *J. High Energy Phys.* **03** (2002) 052 [hep-th/0111025].

[10] C. Herdeiro, S. Hirano and R. Kallosh, *String theory and hybrid inflation/acceleration*, *J. High Energy Phys.* **12** (2001) 027 [hep-th/0110271];
K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, *D3/D7 inflationary model and M-theory*, *Phys. Rev.* **D 65** (2002) 126002 [hep-th/0203019].

[11] J. Garcia-Bellido, R. Rabadán and F. Zamora, *Inflationary scenarios from branes at angles*, *J. High Energy Phys.* **01** (2002) 036 [hep-th/0112147];
N. Jones, H. Stoica and S.H.H. Tye, *Brane interaction as the origin of inflation*, *J. High Energy Phys.* **07** (2002) 051 [hep-th/0203163];
M. Gomez-Reino and I. Zavala, *Recombination of intersecting D-branes and cosmological inflation*, *J. High Energy Phys.* **09** (2002) 020 [hep-th/0207278].

[12] For reviews with many references see: F. Quevedo, *Lectures on string/brane cosmology*, *Class. and Quant. Grav.* **19** (2002) 5721 [hep-th/0210292];
A. Linde, *Prospects of inflation*, hep-th/0402051.

[13] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055].

[14] J.P. Hsu, R. Kallosh and S. Prokushkin, *On brane inflation with volume stabilization*, *JCAP* **12** (2003) 009 [hep-th/0311077];
F. Koyama, Y. Tachikawa and T. Watari, *Supergravity analysis of hybrid inflation model from D3-D7 system*, *Phys. Rev.* **D 69** (2004) 106001 [hep-th/0311191];
H. Firouzjahi and S.H.H. Tye, *Closer towards inflation in string theory*, *Phys. Lett.* **B 584** (2004) 147 [hep-th/0312020];
J.P. Hsu and R. Kallosh, *Volume stabilization and the origin of the inflaton shift symmetry in string theory*, *J. High Energy Phys.* **04** (2004) 042 [hep-th/0402047].

[15] C.P. Burgess, J.M. Cline, H. Stoica and F. Quevedo, *Inflation in realistic D-brane models*, *J. High Energy Phys.* **09** (2004) 033 [hep-th/0403119].

[16] O. DeWolfe, S. Kachru and H. Verlinde, *The giant inflaton*, *J. High Energy Phys.* **05** (2004) 017 [hep-th/0403123];
N. Iizuka and S.P. Trivedi, *An inflationary model in string theory*, *Phys. Rev.* **D 70** (2004) 043519 [hep-th/0403203];
A. Buchel and A. Ghodsi, *Braneworld inflation*, hep-th/0404151.

[17] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[18] See for instance: E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart and D. Wands, *False vacuum inflation with Einstein gravity*, *Phys. Rev.* **D 49** (1994) 6410 [astro-ph/9401011].

[19] C. Escoda, M. Gomez-Reino and F. Quevedo, *Saltatory de Sitter string vacua*, *J. High Energy Phys.* **11** (2003) 065 [hep-th/0307160].

[20] A.D. Linde, *Monopoles as big as a universe*, *Phys. Lett.* **B 327** (1994) 208 [astro-ph/9402031];
A.D. Linde and D.A. Linde, *Topological defects as seeds for eternal inflation*, *Phys. Rev.* **D 50** (1994) 2456 [hep-th/9402115].

[21] A. Vilenkin, *Topological inflation*, *Phys. Rev. Lett.* **72** (1994) 3137 [hep-th/9402085].

[22] K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation with pseudo Nambu-Goldstone bosons*, *Phys. Rev. Lett.* **65** (1990) 3233;
F.C. Adams, J.R. Bond, K. Freese, J.A. Frieman and A.V. Olinto, *Natural inflation: particle physics models, power law spectra for large scale structure and constraints from cobe*, *Phys. Rev.* **D 47** (1993) 426 [hep-ph/9207245].

[23] A.D. Linde, *Initial conditions for inflation*, *Phys. Lett.* **B 162** (1985) 281.

[24] H.V. Peiris et al., *First year wilkinson microwave anisotropy probe (WMAP) observations: implications for inflation*, *Astrophys. J. Suppl.* **148** (2003) 213 [astro-ph/0302225].

[25] A.D. Linde, *Particle Physics and Inflationary Cosmology*, Harwood Academic Publs., Chur 1990.

[26] A.D. Linde, *Eternally existing selfreproducing chaotic inflationary universe*, *Phys. Lett.* **B 175** (1986) 395.

JHEP11(2004)063

[27] A.D. Linde, D.A. Linde and A. Mezhlumian, *From the big bang theory to the theory of a stationary universe*, *Phys. Rev.* **D 49** (1994) 1783 [gr-qc/9306035].

[28] P.J. Steinhardt, *Natural inflation*, in *The very early universe*, G.W. Gibbons, S.W. Hawking and S. Siklos eds., Cambridge University Press, Cambridge 1983;
A.D. Linde, *Nonsingular regenerating inflationary universe*, Cambridge University preprint Print-82-0554 (1982);
A. Vilenkin, *The birth of inflationary universes*, *Phys. Rev.* **D 27** (1983) 2848.

[29] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[30] S. Sethi, C. Vafa and E. Witten, *Constraints on low-dimensional string compactifications*, *Nucl. Phys.* **B 480** (1996) 213 [hep-th/9606122];
K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *J. High Energy Phys.* **08** (1999) 023 [hep-th/9908088].

[31] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, *Naturally vanishing cosmological constant in $N = 1$ supergravity*, *Phys. Lett.* **B 133** (1983) 61;
J.R. Ellis, A.B. Lahanas, D.V. Nanopoulos and K. Tamvakis, *No-scale supersymmetric standard model*, *Phys. Lett.* **B 134** (1984) 429.

[32] C.P. Burgess, R. Kallosh and F. Quevedo, *de Sitter string vacua from supersymmetric D-terms*, *J. High Energy Phys.* **10** (2003) 056 [hep-th/0309187].

[33] R. Brustein and S.P. de Alwis, *Moduli potentials in string compactifications with fluxes: mapping the discretuum*, *Phys. Rev.* **D 69** (2004) 126006 [hep-th/0402088];
A. Saltman and E. Silverstein, *The scaling of the no-scale potential and de Sitter model building*, *J. High Energy Phys.* **11** (2004) 066 [hep-th/0402135].

[34] L. Gorlich, S. Kachru, P. Tripathy and S. Trivedi, *Gaugino condensation and nonperturbative superpotentials in F-theory*, to appear.

[35] F. Denef, M.R. Douglas and B. Florea, *Building a better racetrack*, *J. High Energy Phys.* **06** (2004) 034 [hep-th/0404257].

[36] D. Robbins and S. Sethi, *A barren landscape*, hep-th/0405011.

[37] C.P. Burgess, J.P. Derendinger, F. Quevedo and M. Quiros, *Gaugino condensates and chiral linear duality: an effective lagrangian analysis*, *Phys. Lett.* **B 348** (1995) 428 [hep-th/9501065]; *on gaugino condensation with field-dependent gauge couplings*, *Ann. Phys. (NY)* **250** (1996) 193 [hep-th/9505171].

[38] C.P. Burgess, A. de la Macorra, I. Maksymyk and F. Quevedo, *Supersymmetric models with product groups and field dependent gauge couplings*, *J. High Energy Phys.* **09** (1998) 007 [hep-th/9808087].

[39] E. Cremmer et al., *Spontaneous symmetry breaking and Higgs effect in supergravity without cosmological constant*, *Nucl. Phys.* **B 147** (1979) 105.

[40] E. Witten, *Dimensional reduction of superstring models*, *Phys. Lett.* **B 155** (1985) 151;
C.P. Burgess, A. Font and F. Quevedo, *Low-energy effective action for the superstring*, *Nucl. Phys.* **B 272** (1986) 661.

[41] S. Kachru, J. Pearson and H. Verlinde, *Brane/flux annihilation and the string dual of a non-supersymmetric field theory*, *J. High Energy Phys.* **06** (2002) 021 [hep-th/0112197].

JHEP11(2004)063

[42] R. Kallosh, A. Linde, S. Prokushkin and M. Shmakova, *Supergravity, dark energy and the fate of the universe*, *Phys. Rev.* **D 66** (2002) 123503 [hep-th/0208156].

[43] R. Kallosh and S. Prokushkin, *Supercosmology*, hep-th/0403060.

[44] C.P. Burgess, P. Grenier and D. Hoover, *Quintessentially flat scalar potentials*, *JCAP* **03** (2004) 008 [hep-ph/0308252].

[45] J.F.G. Cascales, M.P. Garcia del Moral, F. Quevedo and A.M. Uranga, *Realistic D-brane models on warped throats: fluxes, hierarchies and moduli stabilization*, *J. High Energy Phys.* **02** (2004) 031 [hep-th/0312051].

[46] L. Kofman, A.D. Linde and A.A. Starobinsky, *Towards the theory of reheating after inflation*, *Phys. Rev.* **D 56** (1997) 3258 [hep-ph/9704452].

[47] U. Seljak et al., *Cosmological parameter analysis including SDSS Ly-alpha forest and galaxy bias: constraints on the primordial spectrum of fluctuations, neutrino mass and dark energy*, astro-ph/0407372.

[48] R. Brustein and P.J. Steinhardt, *Challenges for superstring cosmology*, *Phys. Lett.* **B 302** (1993) 196 [hep-th/9212049].

[49] A.D. Linde, *Quantum creation of the inflationary universe*, *Lett. Nuovo Cim.* **39** (1984) 401.

[50] A. Vilenkin, *Quantum creation of universes*, *Phys. Rev.* **D 30** (1984) 509.

[51] A.D. Linde, *Quantum creation of an open inflationary universe*, *Phys. Rev.* **D 58** (1998) 083514 [gr-qc/9802038].

[52] S.W. Hawking and N. Turok, *Open inflation without false vacua*, *Phys. Lett.* **B 425** (1998) 25 [hep-th/9802030].

[53] A. Linde, *Can we have inflation with $\Omega > 1$?*, *JCAP* **05** (2003) 002 [astro-ph/0303245]; to be published.

[54] D.D. Sokolov and V.F. Shvartsman, *Sov. Phys. JETP* **39** (1975) 196;
G. Paal, *Acta. Phys. Acad. Scient. Hungaricae* **30** (1971) 51;
J.R. Gott, *Mon. Not. R. Astron. Soc.* **193** (1980) 153.

[55] D.D. Sokolov and A.A. Starobinsky, *Sov. Astron.* **19** (1976) 629;
J. Levin, E. Scannapieco and J. Silk, *The topology of the universe: the biggest manifold of them all*, *Class. and Quant. Grav.* **15** (1998) 2689 [gr-qc/9803026];
J. Levin, *Topology and the cosmic microwave background*, *Phys. Rept.* **365** (2002) 251 [gr-qc/0108043];
N.J. Cornish, D.N. Spergel, G.D. Starkman and E. Komatsu, *Constraining the topology of the universe*, *Phys. Rev. Lett.* **92** (2004) 201302 [astro-ph/0310233];
A. Riazuelo, J. Weeks, J.-P. Uzan, R. Lehoucq and J.-P. Luminet, *Cosmic microwave background anisotropies in multi-connected flat spaces*, *Phys. Rev.* **D 69** (2004) 103518 [astro-ph/0311314].

[56] R.H. Brandenberger and C. Vafa, *Superstrings in the early universe*, *Nucl. Phys.* **B 316** (1989) 391.

[57] N.J. Cornish, D.N. Spergel and G.D. Starkman, *Does chaotic mixing facilitate $\Omega < 1$ inflation?*, *Phys. Rev. Lett.* **77** (1996) 215 [astro-ph/9601034].

[58] Y.B. Zeldovich and A.A. Starobinsky, *Quantum creation of a universe in a nontrivial topology*, *Sov. Astron. Lett.* **10** (1984) 135.

JHEP11(2004)063

[59] A. Linde, *Prospects of inflation*, hep-th/0402051.

[60] A. Linde, *Inflation, quantum cosmology and the anthropic principle*, hep-th/0211048.

[61] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *J. High Energy Phys.* **06** (2000) 006 [hep-th/0004134].

[62] L. Susskind, *The anthropic landscape of string theory*, hep-th/0302219.

[63] W.A. Bardeen, S. Elitzur, Y. Frishman and E. Rabinovici, *Fractional charges: global and local aspects*, *Nucl. Phys.* **B 218** (1983) 445;
T. Banks, M. Dine and N. Seiberg, *Irrational axions as a solution of the strong CP problem in an eternal universe*, *Phys. Lett.* **B 273** (1991) 105 [hep-th/9109040].

[64] E.J. Martinec and G.W. Moore, *Noncommutative solitons on orbifolds*, hep-th/0101199.

[65] S. Kachru, R. Kallosh and A. Linde, *String inflation update*, Work in progress.

[66] S. Sarangi and S.H.H. Tye, *Cosmic string production towards the end of brane inflation*, *Phys. Lett.* **B 536** (2002) 185 [hep-th/0204074].

[67] G. Dvali, R. Kallosh and A. Van Proeyen, *D-term strings*, *J. High Energy Phys.* **01** (2004) 035 [hep-th/0312005];
G. Dvali and A. Vilenkin, *Formation and evolution of cosmic D-strings*, *JCAP* **03** (2004) 010 [hep-th/0312007];
E.J. Copeland, R.C. Myers and J. Polchinski, *Cosmic F- and D-strings*, *J. High Energy Phys.* **06** (2004) 013 [hep-th/0312067];
L. Leblond and S.H.H. Tye, *Stability of D1-strings inside a D3-brane*, *J. High Energy Phys.* **03** (2004) 055 [hep-th/0402072];
K. Dasgupta, J.P. Hsu, R. Kallosh, A. Linde and M. Zagermann, *D3/D7 brane inflation and semilocal strings*, *J. High Energy Phys.* **08** (2004) 030 [hep-th/0405247].

[68] G.D. Coughlan, W. Fischler, E.W. Kolb, S. Raby and G.G. Ross, *Cosmological problems for the polonyi potential*, *Phys. Lett.* **B 131** (1983) 59;
T. Banks, D.B. Kaplan and A.E. Nelson, *Cosmological implications of dynamical supersymmetry breaking*, *Phys. Rev.* **D 49** (1994) 779 [hep-ph/9308292];
B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, *Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-D strings*, *Phys. Lett.* **B 318** (1993) 447 [hep-ph/9308325].

[69] J.A. Adams, G.G. Ross and S. Sarkar, *Multiple inflation*, *Nucl. Phys.* **B 503** (1997) 405 [hep-ph/9704286];
G. German, G.G. Ross and S. Sarkar, *Implementing quadratic supergravity inflation*, *Phys. Lett.* **B 469** (1999) 46 [hep-ph/9908380];
G. German, G.G. Ross and S. Sarkar, *Low-scale inflation*, *Nucl. Phys.* **B 608** (2001) 423 [hep-ph/0103243].

JHEP11(2004)063